

The Greek Letters

Chapter 17

Example

- A bank has sold for \$300,000 a European call option on 100,000 shares of a non-dividend paying stock
- $S_0 = 49$, $K = 50$, $r = 5\%$, $\sigma = 20\%$, $T = 20$ weeks, $\mu = 13\%$
- The Black-Scholes value of the option is \$240,000
- How does the bank hedge its risk to lock in a \$60,000 profit?

Naked & Covered Positions

Naked position

Take no action

Covered position

Buy 100,000 shares today

Both strategies leave the bank exposed to significant risk

Stop-Loss Strategy

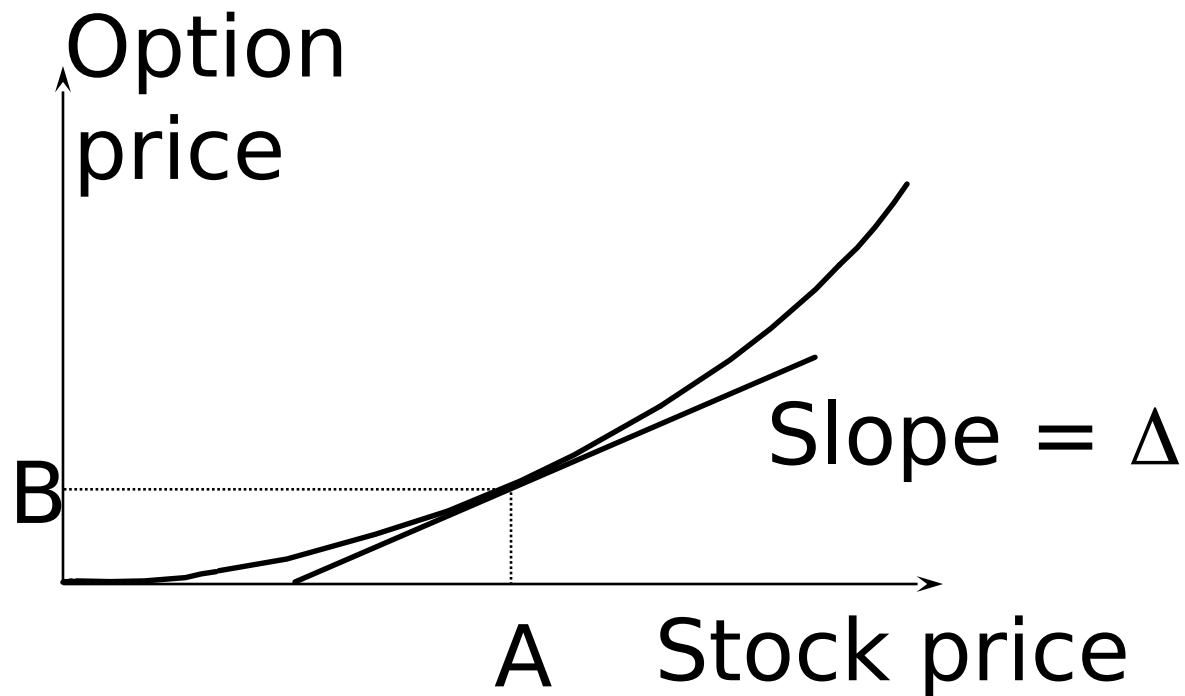
This involves:

- Buying 100,000 shares as soon as price reaches \$50
- Selling 100,000 shares as soon as price falls below \$50

This deceptively simple hedging strategy does not work well

Delta (See Figure 17.2, page 361)

- Delta (Δ) is the rate of change of the option price with respect to the underlying



Delta Hedging

- This involves maintaining a delta neutral portfolio
- The delta of a European call on a non-dividend paying stock is $N(d_1)$
- The delta of a European put on the stock is

$$N(d_1) - 1$$

Delta Hedging continued

- The hedge position must be frequently rebalanced
- Delta hedging a written option involves a “buy high, sell low” trading rule
- See Tables 17.2 (page 364) and 17.3 (page 365) for examples of delta hedging

Theta

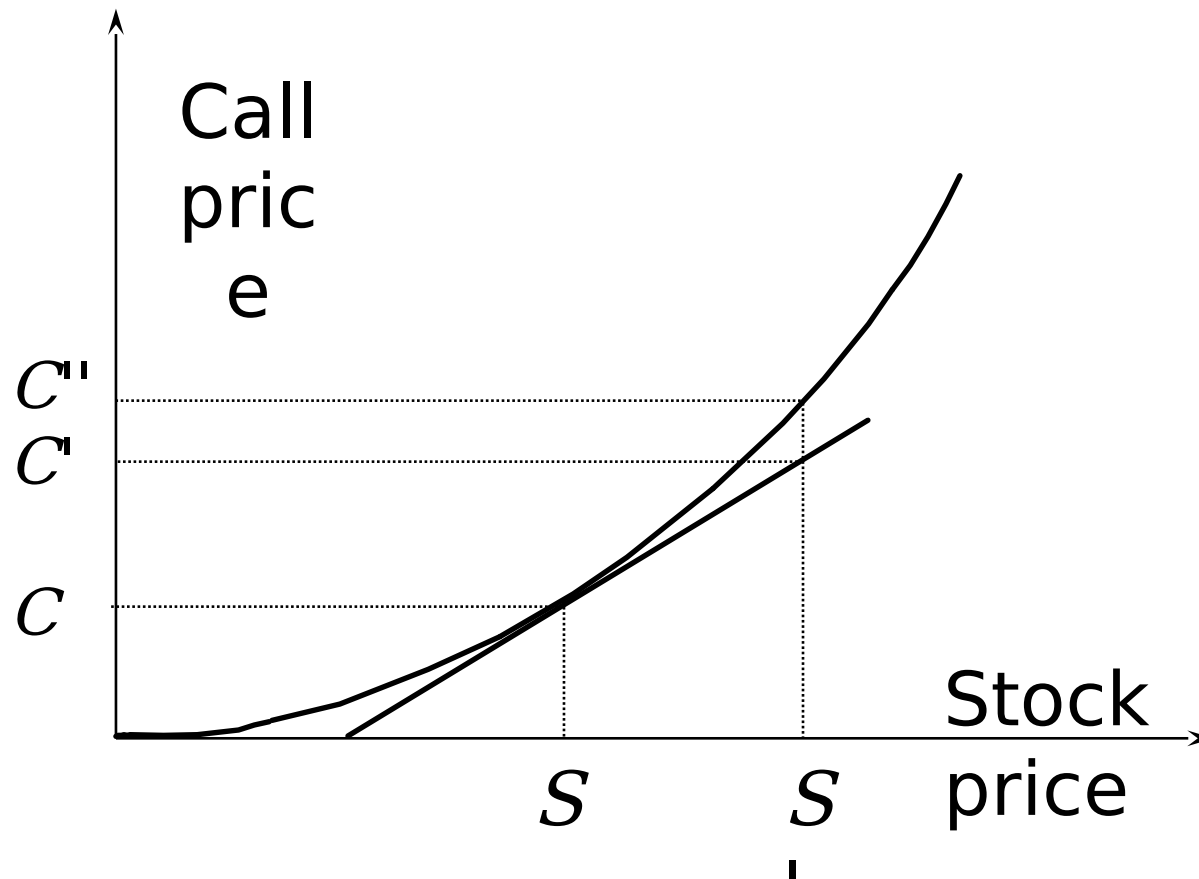
- Theta (Θ) of a derivative (or portfolio of derivatives) is the rate of change of the value with respect to the passage of time
- The theta of a call or put is usually negative. This means that, if time passes with the price of the underlying asset and its volatility remaining the same, the value of a long option declines
- See Figure 17.5 for the variation of Θ with respect to the stock price for a European call

Gamma

- Gamma (Γ) is the rate of change of delta (Δ) with respect to the price of the underlying asset
- Gamma is greatest for options that are close to the money (see Figure 17.9, page 372)

Gamma Addresses Delta Hedging Errors Caused By Curvature

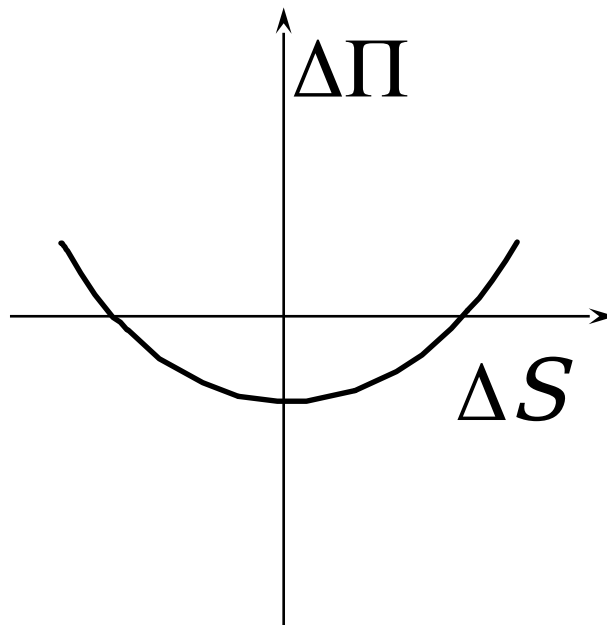
(Figure 17.7, page 369)



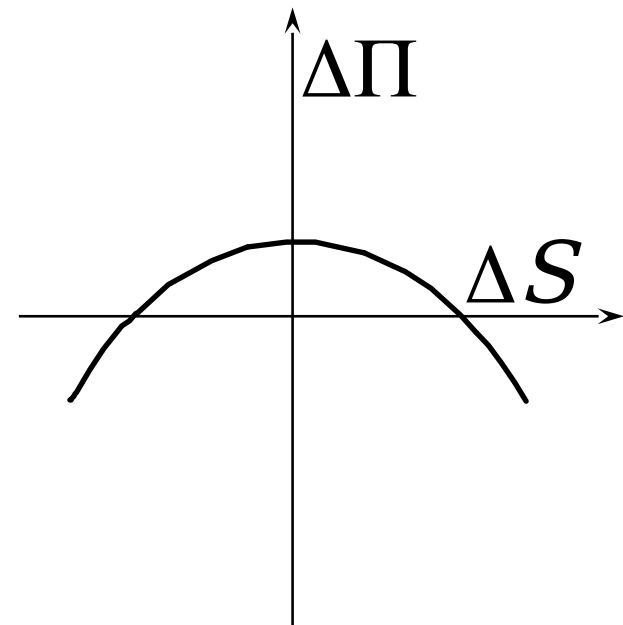
Interpretation of Gamma

- For a delta neutral portfolio,

$$\Delta\Pi \approx \Theta \Delta t + \frac{1}{2}\Gamma\Delta S^2$$



Positive Gamma



Negative Gamma

Relationship Between Delta, Gamma, and Theta (page 373)

For a portfolio of derivatives on a stock paying a continuous dividend yield at rate q

$$\Theta + rS\Delta + \frac{1}{2}\sigma^2 S^2\Gamma = r\Pi$$

Vega

- Vega ($\mathcal{V}$) is the rate of change of the value of a derivatives portfolio with respect to volatility
- Vega tends to be greatest for options that are close to the money (See Figure 17.11, page 374)

Managing Delta, Gamma, & Vega

- Δ can be changed by taking a position in the underlying
- To adjust Γ & v it is necessary to take a position in an option or other derivative

Rho

- Rho is the rate of change of the value of a derivative with respect to the interest rate
- For currency options there are 2 rhos

Hedging in Practice

- Traders usually ensure that their portfolios are delta-neutral at least once a day
- Whenever the opportunity arises, they improve gamma and vega
- As portfolio becomes larger hedging becomes less expensive

Scenario Analysis

A scenario analysis involves testing the effect on the value of a portfolio of different assumptions concerning asset prices and their volatilities

Greek Letters for Options on an Asset that Provides a Dividend Yield at Rate q

- See Table 17.6 on page 378

Futures Contract Can Be Used for Hedging

- The delta of a futures contract on an asset paying a yield at rate q is $e^{(r-q)T}$ times the delta of a spot contract
- The position required in futures for delta hedging is therefore $e^{-(r-q)T}$ times the position required in the corresponding spot contract

Hedging vs Creation of an Option Synthetically

- When we are hedging we take positions that offset Δ , Γ , ν , etc.
- When we create an option synthetically we take positions that match Δ , Γ , & ν

Portfolio Insurance

- In October of 1987 many portfolio managers attempted to create a put option on a portfolio synthetically
- This involves initially selling enough of the portfolio (or of index futures) to match the Δ of the put option

Portfolio Insurance continued

- As the value of the portfolio increases, the Δ of the put becomes less negative and some of the original portfolio is repurchased
- As the value of the portfolio decreases, the Δ of the put becomes more negative and more of the portfolio must be sold

Portfolio Insurance continued

The strategy did not work well on
October 19, 1987...